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MAT 112B Midterm Exam Review

We are very glad you've joined us! We'll discuss how to use "student learning outcomes" to help you study, then work through any problems you have questions over in the packet provided. If you let us know how we did via an email survey (sent out in 1-2 days), you'll be automatically entered to win a prize, to be announced via your ECU.edu email account.

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What are Student Learning Outcomes?

Student learning outcomes state what a student is expected to know or be able to do after completing a section of material. Learning outcomes range from simple, concrete tasks, such as remembering facts, defining terms, or recognizing a concept, listed at the bottom of the taxonomy, all the way to the complex, abstract tasks of creating new or original work, based on knowledge gained from the learning activity.

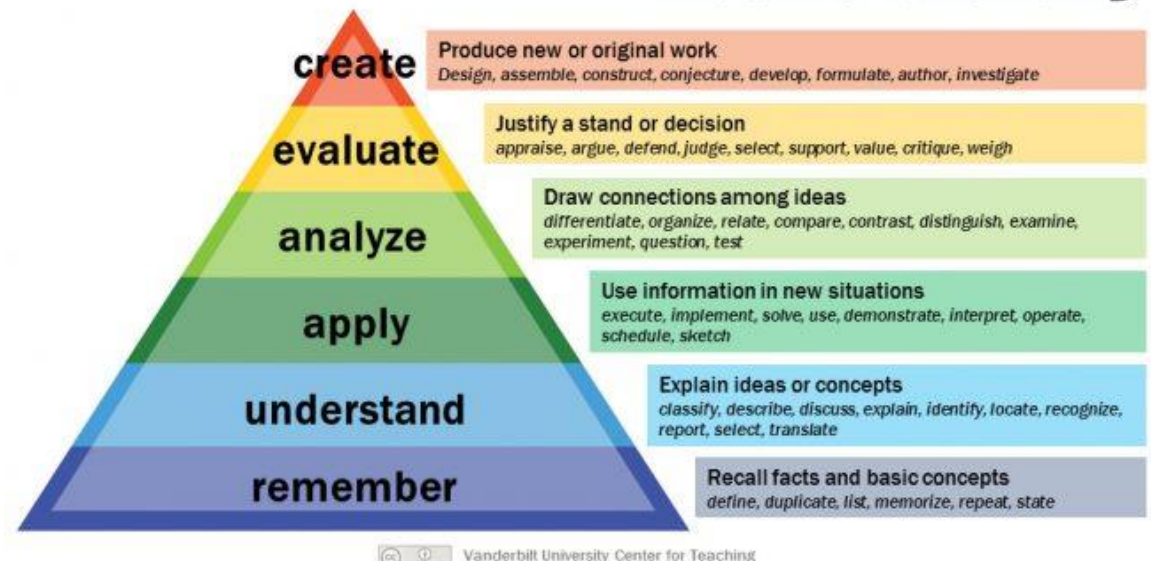
Most textbooks, including yours, list the learning outcomes to help guide your studies! To use the learning outcomes to your advantage, take each one and turn them into questions, asking yourself:

1. Do I recognize all the terms in the question?
2. Do I understand what the question is asking me to do?
3. Can I find examples of questions that match this learning outcome?
4. Can I answer the question?
5. Do I know how to check my work?

If you answer "no" to any of these questions, you know where to focus your efforts!

Screenshots of questions, images, and text on the following pages are from *College Algebra in Context*, 6th edition, by R.J. Harshbarger and L.S. Yocco; Hoboken, NJ: Pearson (2021)

Bloom's Taxonomy

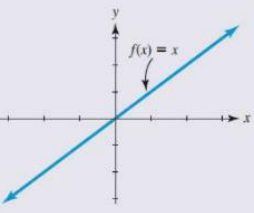
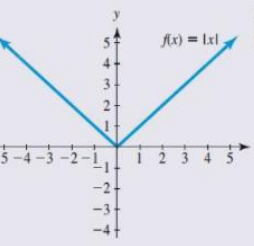
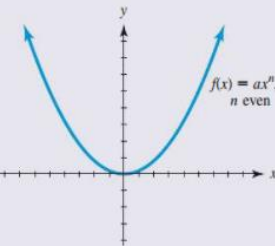
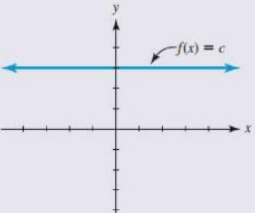
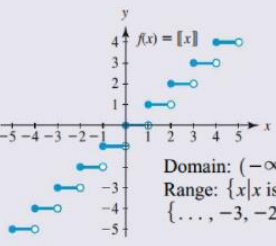
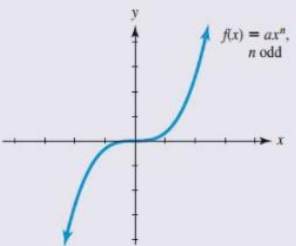


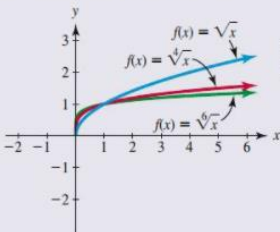
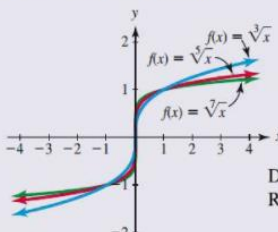
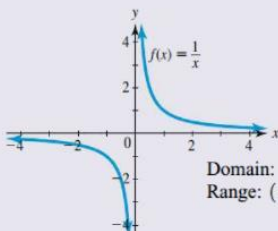
Vanderbilt University Center for Teaching

<https://cft.vanderbilt.edu/guides-sub-pages/blooms-taxonomy/#2001>

Chapter 4: Additional Topics with Functions

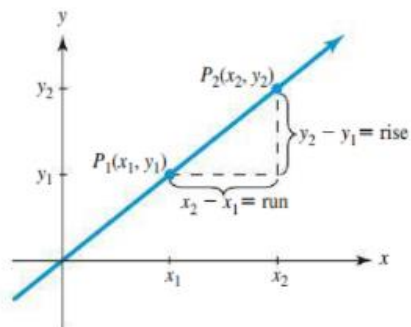
Chapter 4 Algebra Toolbox

Learning Outcomes and Notes	Examples
Identify the domain and range of a given type of function. IDENTITY FUNCTION  Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ - Increases on its domain ABSOLUTE VALUE FUNCTION  Domain: $(-\infty, \infty)$ Range: $[0, \infty)$ - Decreases on $(-\infty, 0)$, increases on $(0, \infty)$ POWER FUNCTION, n EVEN  - Decreases on $(-\infty, 0)$, increases on $(0, \infty)$ for $a > 0$ - Increases on $(-\infty, 0)$, decreases on $(0, \infty)$ for $a < 0$ CONSTANT FUNCTION  Domain: $(-\infty, \infty)$ Range: $\{c\}$, where c is a constant - Graph is a horizontal line. GREATEST INTEGER FUNCTION  Domain: $(-\infty, \infty)$ Range: $\{x x \text{ is an integer}\} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ $f(x) = [x]$ is a special step function. POWER FUNCTION, n ODD  - Increases on $(-\infty, \infty)$ for $a > 0$ - Decreases on $(-\infty, \infty)$ for $a < 0$	- The domain of the reciprocal function is _____, and its range is _____. - The domain of the constant function $g(x) = k$ is _____, and its range is _____. - The range of the squaring function is _____. - The domain of the squaring function is _____.

ROOT FUNCTION, n EVEN  Domain: $[0, \infty)$ Range: $[0, \infty)$ Increases on its domain	ROOT FUNCTION, n ODD  Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ - Increases on its domain RECIPROCAL FUNCTION  Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$ - Decreases on $(-\infty, 0)$ and $(0, \infty)$	
Determine over what intervals a function is increasing or decreasing.	3. The reciprocal function decreases on _____. 4. The absolute value function increases on the interval _____ and decreases on _____.	
Determine if a function is increasing or decreasing over given intervals.	<i>In Exercises 7–12, determine whether the function is increasing or decreasing on the given interval.</i> 7. $g(x) = \sqrt[5]{x}; (-\infty, \infty)$ 8. $h(x) = \sqrt[4]{x}; [0, \infty)$ 9. $f(x) = 5 - 0.8x; (-\infty, \infty)$ 10. $f(x) = \frac{-1}{x}; (-\infty, 0)$ 11. $g(x) = -2x^{2/3}; (0, \infty)$ 12. $h(x) = 3x^{1/6}; [0, \infty)$	

Linear Functions

$$y = mx + b$$



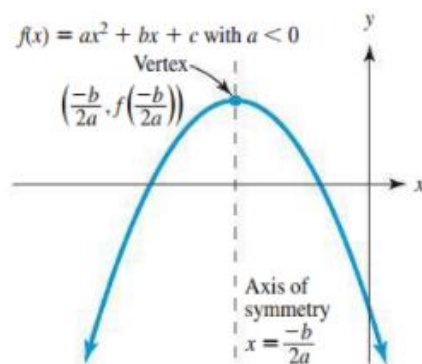
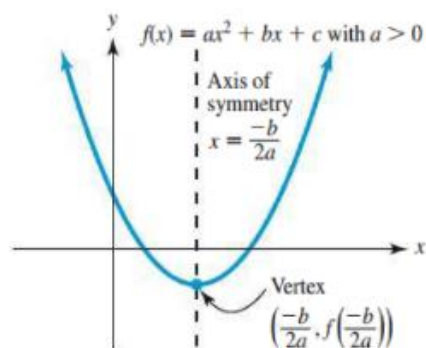
- m is the slope of the line, b is the y -intercept:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- The function is increasing if $m > 0$.
- The function is decreasing if $m < 0$.

Identify a function from its equation and graph it.

Quadratic Functions



- Vertex is at the point $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

Identify the type of each of the following functions from its equation. Then graph the function.

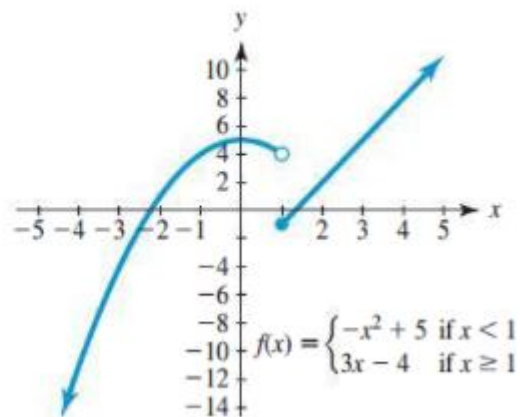
13. $f(x) = x^3$

14. $f(x) = \sqrt{x}$

15. $f(x) = 8\sqrt[6]{x}$

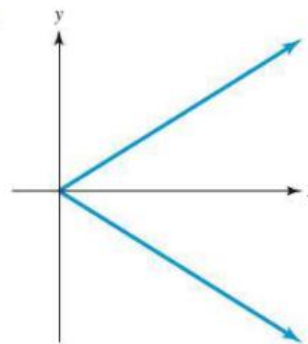
16. $f(x) = \frac{-4}{x^3}$

Piecewise-Defined Functions

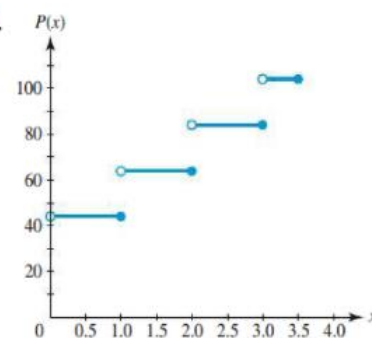


Determine if a given graph represents a function.

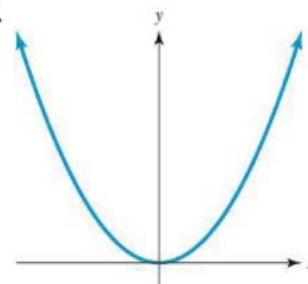
17.



21.



22.



4.1 Transformation of Graphs and Symmetry

Learning Outcomes and Notes

Graph related functions and describe the transformations used to obtain one from another.

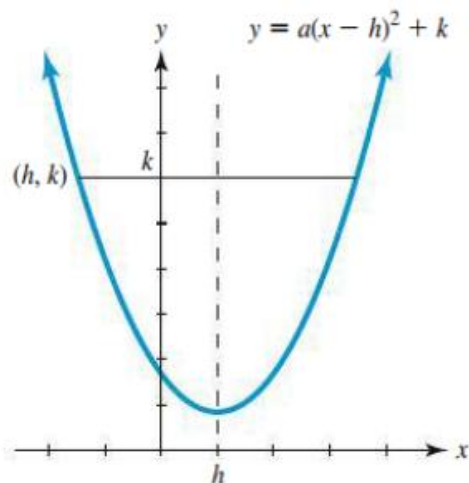


Figure 4.4

Vertical Shifts of Graphs

If k is a positive real number:

The graph of $g(x) = f(x) + k$ can be obtained by shifting the graph of $f(x)$ upward k units.

The graph of $g(x) = f(x) - k$ can be obtained by shifting the graph of $f(x)$ downward k units.

Horizontal Shifts of Graphs

If h is a positive real number:

The graph of $g(x) = f(x - h)$ can be obtained by shifting the graph of $f(x)$ to the right h units.

The graph of $g(x) = f(x + h)$ can be obtained by shifting the graph of $f(x)$ to the left h units.

Examples

In Exercises 1–16, (a) sketch the graph of each pair of functions using a standard window, and (b) describe the transformations used to obtain the graph of the second function from the first function.

1. $y = x^3, y = x^3 + 5$
2. $y = x^2, y = x^2 + 3$
3. $y = \sqrt{x}, y = \sqrt{x - 4}$
4. $y = x^2, y = (x + 2)^2$
5. $y = \sqrt[3]{x}, y = \sqrt[3]{x + 2} - 1$
6. $y = x^3, y = (x - 5)^3 - 3$
7. $y = |x|, y = |x - 2| + 1$
8. $y = |x|, y = |x + 3| - 4$
9. $y = x^2, y = -x^2 + 5$
10. $y = \sqrt{x}, y = -\sqrt{x - 2}$
11. $y = \frac{1}{x}, y = \frac{1}{x} - 3$
12. $y = \frac{1}{x}, y = \frac{2}{x - 1}$
13. $f(x) = x^2, g(x) = \frac{1}{3}x^2$

Stretching and Compressing Graphs

The graph of $y = af(x)$ is obtained by vertically stretching the graph of $f(x)$ using a factor of $|a|$ if $|a| > 1$ and vertically compressing the graph of $f(x)$ using a factor of $|a|$ if $0 < |a| < 1$.

Reflections of Graphs Across the Coordinate Axes

1. The graph of $y = -f(x)$ can be obtained by reflecting the graph of $y = f(x)$ across the x -axis.
2. The graph of $y = f(-x)$ can be obtained by reflecting the graph of $y = f(x)$ across the y -axis.

Graph Transformations

For a given function $y = f(x)$,

Vertical Shift	$y = f(x) + k$	Graph is shifted k units up if $k > 0$ and k units down if $k < 0$.
Horizontal Shift	$y = f(x - h)$	Graph is shifted h units right if $h > 0$ and h units left if $h < 0$.
Stretch/Compress	$y = af(x)$	Graph is vertically stretched using a factor of $ a $ if $ a > 1$. Graph is compressed using a factor of $ a $ if $ a < 1$.
Reflection	$y = -f(x)$ $y = f(-x)$	Graph is reflected across the x -axis. Graph is reflected across the y -axis.

Symmetry with Respect to the y -axis

The graph of $y = f(x)$ is symmetric with respect to the y -axis if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph—that is,

$$f(-x) = f(x)$$

for all x in the domain of f . Such a function is called an **even function**.

Symmetry with Respect to the Origin

The graph of $y = f(x)$ is symmetric with respect to the origin if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph—that is,

$$f(-x) = -f(x)$$

for all x in the domain of f . Such a function is called an **odd function**.

14. $f(x) = x^3, g(x) = 0.4x^3$

15. $f(x) = |x|, g(x) = 3|x|$

16. $f(x) = \sqrt{x}, g(x) = 4\sqrt{x}$

17. How is the graph of $y = (x - 2)^2 + 3$ transformed from the graph of $y = x^2$?

18. How is the graph of $y = (x + 4)^3 - 2$ transformed from the graph of $y = x^3$?

19. Suppose the graph of $y = x^{3/2}$ is shifted to the left 4 units. What is the equation that gives the new graph?

20. Suppose the graph of $y = x^{3/2}$ is shifted down 5 units and to the right 4 units. What is the equation that gives the new graph?

21. Suppose the graph of $y = x^{3/2}$ is stretched by a factor of 3 and then shifted up 5 units. What is the equation that gives the new graph?

22. Suppose the graph of $y = x^{2/3}$ is compressed by a factor of $\frac{1}{5}$ and then shifted right 6 units. What is the equation that gives the new graph?

23. Suppose the graph of $f(x) = \sqrt{x}$ is shifted up 3 units and to the left 6 units. What is the equation of the new graph? Verify the result graphically.

24. Suppose the graph of $f(x) = x^2$ is shifted up 2 units and to the right 5 units. What is the equation of the new graph? Verify the result graphically.

Symmetry with Respect to the x-axis

The graph of an equation is symmetric with respect to the x -axis if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.

Identify transformations from equations and write equations from given transformations.

Write an equation of a transformed function from its graph.

25. Suppose the graph of $f(x) = \frac{1}{x}$ is stretched vertically by a factor of 4 and then shifted down 3 units. What is the equation of the new graph? Verify the result graphically.

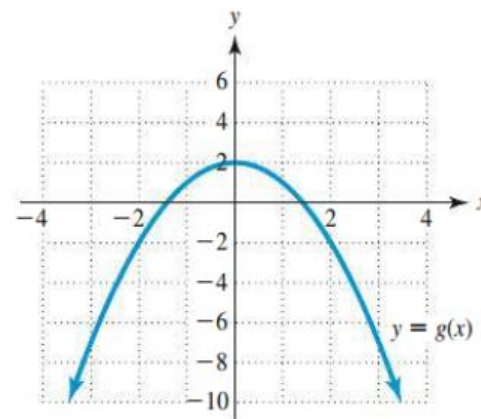
26. Suppose the graph of $f(x) = x^3$ is compressed vertically using a factor of $\frac{1}{3}$ and then shifted to the left 5 units. What is the equation of the new graph? Verify the result graphically.

27. Suppose the graph of $f(x) = \sqrt{x}$ is reflected across the x -axis and shifted up 2 units. What is the equation of the new graph? Verify the result graphically.

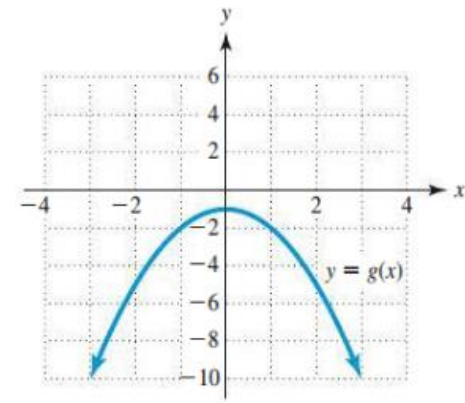
28. Suppose the graph of $f(x) = \sqrt[3]{x}$ is reflected across the x -axis and shifted to the right 1 unit. What is the equation of the new graph? Verify the result graphically.

In Exercises 29–32, write the equation of the function $g(x)$ that is transformed from the given function $f(x)$ and whose graph is shown.

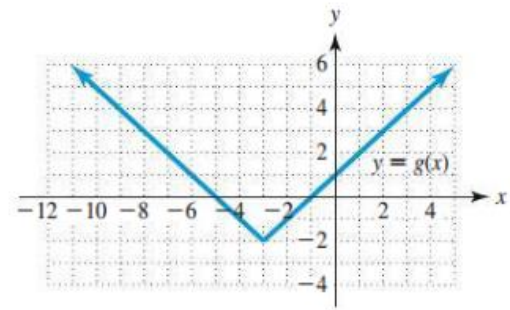
29. $f(x) = x^2$



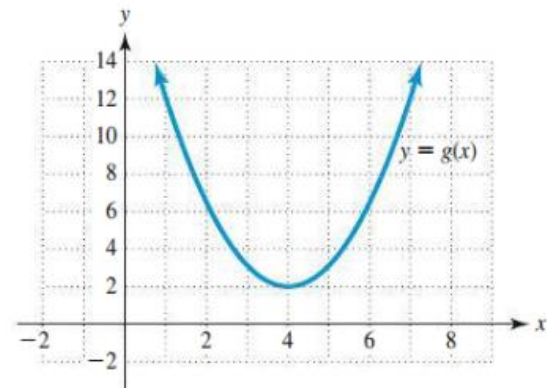
30. $f(x) = x^2$



31. $f(x) = |x|$



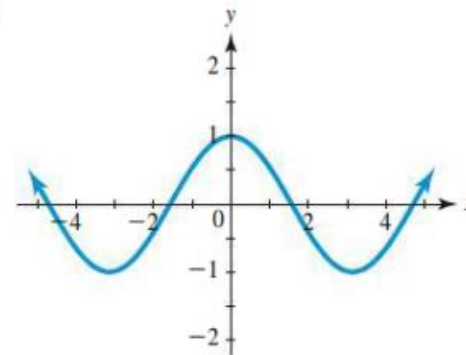
32. $f(x) = x^2$



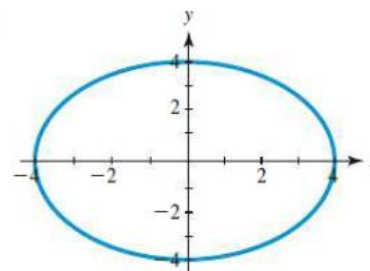
Determine symmetry visually.

In Exercises 33–36, determine visually whether each of the graphs is symmetric about the x -axis, the y -axis, or neither. If the graph represents a function, determine whether the function is even, odd, or neither.

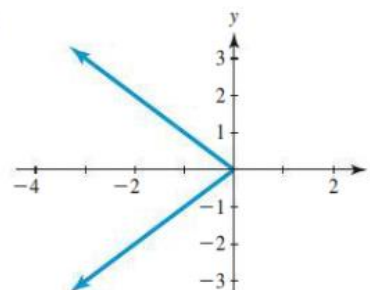
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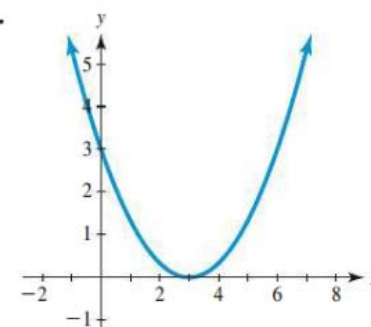
34.



35.



36.



Determine symmetry using a table of ordered pairs.

In Exercises 37 and 38, determine whether the graph of the equation in Y_1 is symmetric about the y-axis, based on the ordered pairs shown in each pair of tables.

37.

NORMAL FLOAT AUTO REAL RADIAN MP					
X	Y ₁				
0	0				
1	2				
2	8				
3	18				
4	32				
5	50				

X=

NORMAL FLOAT AUTO REAL RADIAN MP					
X	Y ₁				
0	0				
-1	2				
-2	8				
-3	18				
-4	32				
-5	50				

X=

38.

NORMAL FLOAT AUTO REAL RADIAN MP					
X	Y ₁				
0	0				
1	.5				
2	2				
3	4.5				
4	8				
5	12.5				

X=

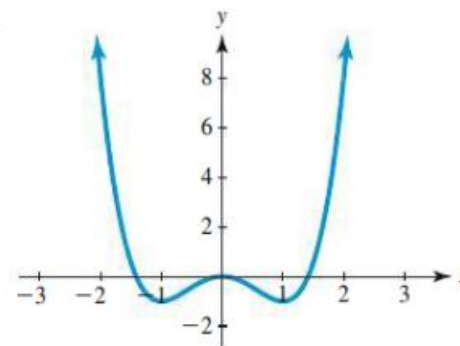
NORMAL FLOAT AUTO REAL RADIAN MP					
X	Y ₁				
0	0				
-1	.5				
-2	2				
-3	4.5				
-4	8				
-5	12.5				

X=0

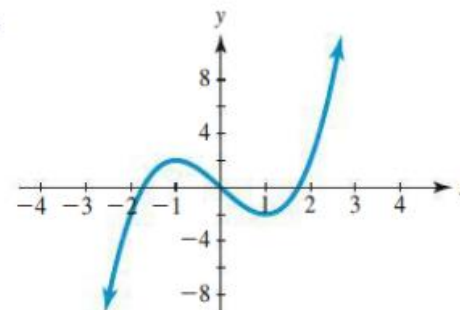
Determine symmetry algebraically.	<i>In Exercises 39–46, determine algebraically whether the graph of the given equation is symmetric with respect to the x-axis, the y-axis, and/or the origin. Confirm graphically.</i> 39. $y = 2x^2 - 3$ 40. $y = -x^2 + 4$ 41. $y = x^3 - x$ 42. $y = -x^3 + 5x$ 43. $y = \frac{6}{x}$ 44. $x = 3y^2$ 45. $x^2 + y^2 = 25$ 46. $x^2 - y^2 = 25$
Determine whether functions are even, odd, or neither.	<i>In Exercises 47–52, determine whether the function is even, odd, or neither.</i> 47. $f(x) = x - 5$ 48. $f(x) = x - 2$ 49. $g(x) = \sqrt{x^2 + 3}$ 50. $f(x) = \frac{1}{2}x^3 - x$ 51. $g(x) = \frac{5}{x}$ 52. $g(x) = 4x + x^2$

Determine whether each of the complete graphs in Exercises 53 and 54 represents a function that is even, odd, or neither.

53.



54.



55. Graph $(y + 1)^2 = x + 4$. Does this graph have any type of symmetry?

56. About what line is the function $y = 3 \pm \sqrt{x - 2}$ symmetric?

Solve applications involving transformations of graphs.	57. Medical Marijuana Revenue Using data from 2016 and projected to 2025, the U.S. medical marijuana revenue, in billions of dollars, can be modeled by the function $M(x) = 0.037(x - 6)^2 + 0.652(x - 6) + 4.537$ where x is the number of years after 2010. (Source: thecannabist.co) - The graph of this function is a shifted graph of which basic function? - Find and interpret $M(24)$. - Graph $y = M(x)$ for x representing the years 2016 through 2025.
Answer questions involving transformation of functions.	59. Supply and Demand The price per unit of a product is $\\$p$, and the number of units of the product is denoted by q. The supply function for a product is given by $p = \frac{180 + q}{6}$, and the demand for the product is given by $p = \frac{30,000}{q} - 20$. - Is the supply function a linear function or a shifted reciprocal function? - Is the demand function a shifted linear function or a shifted reciprocal function? Describe the transformations needed to obtain the specific function from the basic function.

61. Average Cost The monthly average cost of producing x sofas is $\bar{C}(x) = \frac{30,000}{x} + 150$ dollars.

- a. Will the average cost function decrease or increase as the number of units produced increases?
- b. What transformations of the graph of the reciprocal function give the graph of this function?

62. Pollution The daily cost C (in dollars) of removing pollution from the smokestack of a coal-fired electric power plant is related to the percent of pollution p being removed according to the equation

$$C = \frac{10,500}{100 - p}$$

- a. Describe the transformations needed to obtain this function from the function $C = \frac{1}{p}$.
- b. Graph the function for $0 \leq p < 100$.
- c. What is the daily cost of removing 80% of the pollution?

4.2 Combining Functions; Composite Functions

Learning Outcomes and Notes			Examples
Find the sum, difference, product, and quotient of functions from their equations.			<i>In Exercises 1–8, find the following: (a) $(f + g)(x)$, (b) $(f - g)(x)$, (c) $(f \cdot g)(x)$, (d) $\left(\frac{f}{g}\right)(x)$, and (e) the domain of $\frac{f}{g}$.</i>
Operation	Formula	Example with $f(x) = \sqrt{x}$ and $g(x) = x^3$	
Sum	$(f + g)(x) = f(x) + g(x)$	$(f + g)(x) = \sqrt{x} + x^3$	1. $f(x) = 3x - 5; g(x) = 4 - x$
Difference	$(f - g)(x) = f(x) - g(x)$	$(f - g)(x) = \sqrt{x} - x^3$	2. $f(x) = 2x - 3; g(x) = 5 - x$
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$	$(f \cdot g)(x) = \sqrt{x} \cdot x^3 = x^3\sqrt{x}$	3. $f(x) = x^2 - 2x; g(x) = 1 + x$
Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad (g(x) \neq 0)$	$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x^3} \quad (x \neq 0)$	4. $f(x) = 2x^2 - x; g(x) = 2x + 1$
			5. $f(x) = \frac{1}{x}; g(x) = \frac{x + 1}{5}$
			6. $f(x) = \frac{x - 2}{3}; g(x) = \frac{1}{x}$
			7. $f(x) = \sqrt{x}; g(x) = 1 - x^2$
			8. $f(x) = x^3; g(x) = \sqrt{x + 3}$
			9. If $f(x) = x^2 - 5x$ and $g(x) = 6 - x^3$, evaluate
			a. $(f + g)(2)$ b. $(g - f)(-1)$
			c. $(f \cdot g)(-2)$ d. $\left(\frac{g}{f}\right)(3)$
			10. If $f(x) = 4 - x^2$ and $g(x) = x^3 + x$, evaluate
			a. $(f + g)(1)$ b. $(f - g)(-2)$
			c. $(f \cdot g)(-3)$ d. $\left(\frac{g}{f}\right)(2)$

Find the composition of functions from their equations.

Composite Function

The composite function f of g is denoted by $f \circ g$ and defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the subset of the domain of g for which $f \circ g$ is defined.

The composite function $g \circ f$ is defined by $(g \circ f)(x) = g(f(x))$.

The domain of $g \circ f$ is the subset of the domain of f for which $g \circ f$ is defined.

In Exercises 11–20, find (a) $(f \circ g)(x)$ and (b) $(g \circ f)(x)$.

11. $f(x) = 2x - 6$; $g(x) = 3x - 1$

12. $f(x) = 3x - 2$; $g(x) = 2x - 2$

13. $f(x) = x^2$; $g(x) = \frac{1}{x}$ 14. $f(x) = x^3$; $g(x) = \frac{2}{x}$

15. $f(x) = \sqrt{x - 1}$; $g(x) = 2x - 7$

16. $f(x) = \sqrt{3 - x}$; $g(x) = x - 5$

17. $f(x) = |x - 3|$; $g(x) = 4x$

18. $f(x) = |4 - x|$; $g(x) = 2x + 1$

19. $f(x) = \frac{3x + 1}{2}$; $g(x) = \frac{2x - 1}{3}$

20. $f(x) = \sqrt[3]{x + 1}$; $g(x) = x^3 + 1$

In Exercises 21 and 22, use $f(x)$ and $g(x)$ to evaluate each expression.

21. $f(x) = 2x^2$; $g(x) = \frac{x - 5}{3}$

a. $(f \circ g)(2)$

b. $(g \circ f)(-2)$

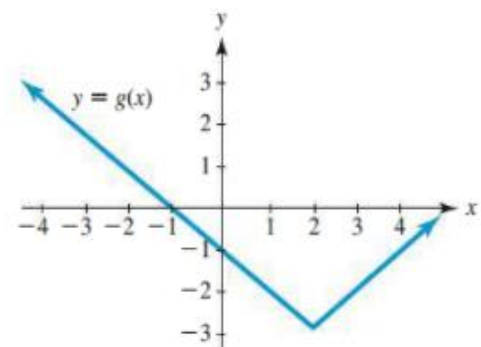
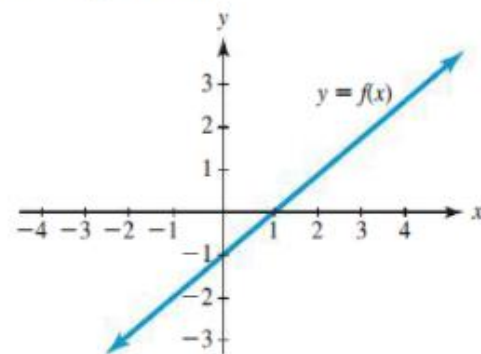
22. $f(x) = (x - 1)^2$; $g(x) = 3x - 1$

a. $(f \circ g)(2)$

b. $(g \circ f)(-2)$

Find compositions, sums, and quotients of functions from their graphs.

In Exercises 23 and 24, use the following graphs of f and g to evaluate the functions.



- | | |
|---|-----------------------------|
| 23. a. $(f + g)(2)$ | b. $(f \circ g)(-1)$ |
| c. $\left(\frac{f}{g}\right)(4)$ | d. $(f \circ g)(1)$ |
| e. $(g \circ f)(-2)$ | |
| 24. a. $(g - f)(-2)$ | b. $(f \circ g)(3)$ |
| c. $\left(\frac{f}{g}\right)(0)$ | d. $(f \circ g)(-2)$ |
| e. $(g \circ f)(2)$ | |

Solve applications involving function operations and composite functions.	25. Profit Suppose that the total weekly cost for the production and sale of x bicycles is $C(x) = 23x + 3420$ dollars and that the total revenue is given by $R(x) = 89x$ dollars, where x is the number of bicycles. - Write the equation of the function that models the weekly profit from the production and sale of x bicycles. - What is the profit on the production and sale of 150 bicycles? - Write the function that gives the average profit per bicycle. - What is the average profit per bicycle if 150 are produced and sold? 28. Revenue and Cost The total monthly revenue function for Easy-Ride golf carts is given by $R = 26,600x$ dollars, and the total monthly cost function for the carts is $C = 200,000 + 4600x + 2x^2$ dollars, where x is the number of golf carts that are produced and sold. - Which function is quadratic, and which is linear? - Form the profit function for the golf carts from these two functions. - Is the profit function a linear function, a quadratic function, or neither of these?
Answer interactive figure questions involving the sum and difference of two functions.	32. Cost-Benefit Suppose that for a certain city the cost C of obtaining drinking water that contains $p\%$ impurities (by volume) is given by $C = \frac{120,000}{p} - 1200$

	a. This function can be considered as the difference of what two functions? b. What is the cost of drinking water that is 80% impure? 52. Wind Chill If the air temperature is 25°F, the wind chill temperature C is given by $C = 59.914 - 2.35s - 20.14\sqrt{s}$, where s is the wind speed in miles per hour. a. State two functions whose difference gives this function. b. Graph this function for $3 \leq s \leq 12$. c. Is the function increasing or decreasing on this domain?
Answer questions involving the product/quotient of two functions.	31. Average Cost If the monthly total cost of producing 27-inch television sets is given by $C(x) = 50,000 + 105x$, where x is the number of sets produced per month, then the average cost per set is given by $\bar{C}(x) = \frac{50,000 + 105x}{x}$ a. Explain how $C(x)$ and another function can be combined to obtain the average cost function. b. What is the average cost per set if 3000 sets are produced?

4.3 One-to-One and Inverse Functions

Learning Outcomes and Notes	Examples
Determine whether a function defined by a relation has an inverse and, if so, find the inverse. Inverse Functions Functions f and g for which $f(g(x)) = x$ for all x in the domain of g and $g(f(x)) = x$ for all x in the domain of f are called inverse functions. In this case, we denote g by f^{-1}, read as “f inverse.”	<i>In Exercises 1 and 2, determine if the function f defined by the arrow diagram has an inverse. If it does, create an arrow diagram that defines the inverse. If it does not, explain why not.</i> 1. Domain Range 2. Domain Range <i>In Exercises 3 and 4, determine whether the function f defined by the set of ordered pairs has an inverse. If it does, find the inverse.</i> 3. $\{(5, 2), (4, 1), (3, 7), (6, 2)\}$ 4. $\{(2, 8), (3, 9), (4, 10), (5, 11)\}$
Determine whether two functions are inverses.	5. If $f(x) = 3x$ and $g(x) = \frac{x}{3}$, - What are $f(g(x))$ and $g(f(x))$? - Are $f(x)$ and $g(x)$ inverse functions? 6. If $f(x) = 4x - 1$ and $g(x) = \frac{x+1}{4}$, - What are $f(g(x))$ and $g(f(x))$? - Are $f(x)$ and $g(x)$ inverse functions? 7. If $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$, are $f(x)$ and $g(x)$ inverse functions? 8. If $f(x) = (x-2)^3$ and $g(x) = \sqrt[3]{x+2}$, are $f(x)$ and $g(x)$ inverse functions?

Evaluate functions and their inverse.

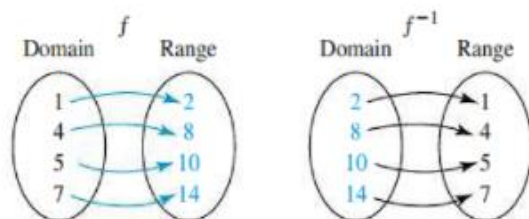


Figure 4.32

9. For the function f defined by $f(x) = 3x - 4$, complete the tables below for f and f^{-1} .

x	$f(x)$	x	$f^{-1}(x)$
-1	-7	-7	-1
0		-4	
1		-1	
2		2	
3		5	

10. For the function g defined by $g(x) = 2x^3 - 1$, complete the tables below for g and g^{-1} .

x	$g(x)$	x	$g^{-1}(x)$
-2	-17	-17	-2
-1		-3	
0		-1	
1		1	
2		15	

Determine whether a function is one-to-one.

One-to-One Function

A function f is a one-to-one function if each output of the function corresponds to exactly one input in the domain of the function. This means there is a one-to-one correspondence between the elements of the domain and the elements of the range.

In Exercises 11–14, determine whether the function is one-to-one.

11. $\{(1, 5), (2, 6), (3, 7), (4, 5)\}$
 12. $\{(2, -4), (5, -8), (8, -12), (11, -16)\}$
 13. $f(x) = (x - 3)^3$ 14. $f(x) = \frac{1}{x}$

Horizontal Line Test

A function is one-to-one if no horizontal line can intersect the graph of the function in more than one point.

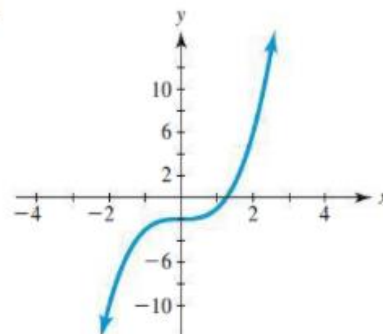
In Exercises 17 and 18, determine whether the function is *one-to-one*.

17. $y = -2x^4$

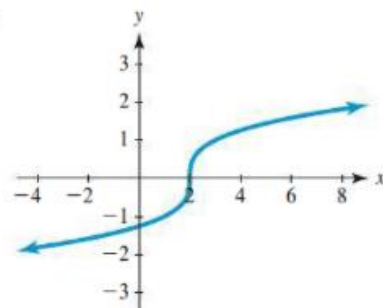
18. $y = \sqrt{x+3}$

In Exercises 15 and 16, determine whether each graph is the graph of a *one-to-one* function.

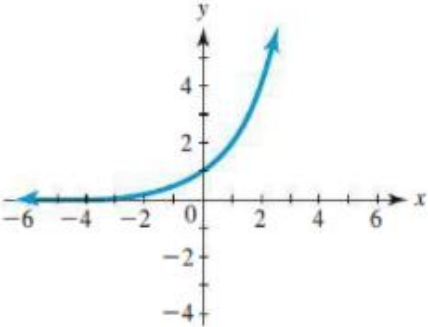
15.



16.



Find the inverse of a function. Inverse Functions The functions f and g are inverse functions if, whenever the pair (a, b) satisfies $y = f(x)$, the pair (b, a) satisfies $y = g(x)$. Note that when this happens, $f(g(x)) = x \quad \text{and} \quad g(f(x)) = x$ for all x in the domain of g and f, respectively. The domain of the function f is the range of its inverse g, and the domain of g is the range of f. Finding the Inverse of a Function To find the inverse of the function f that is defined by the equation $y = f(x)$: 1. Rewrite the equation, replacing $f(x)$ with y. 2. Interchange x and y in the equation defining the function. 3. Solve the new equation for y. If this equation cannot be solved uniquely for y, the original function has no inverse function. 4. Replace y with $f^{-1}(x)$.	19. a. Write the inverse of $f(x) = 3x - 4$. b. Do the values for f^{-1} in the table of Exercise 9 fit the equation for f^{-1}? 20. a. Write the inverse of $g(x) = 2x^3 - 1$. b. Do the values for g^{-1} in the table of Exercise 10 fit the equation for g^{-1}? 21. If function h has an inverse and $h^{-1}(-2) = 3$, find $h(3)$. 22. Find the inverse of $f(x) = \frac{1}{x}$. 23. Find the inverse of $g(x) = 4x + 1$.
Find inverse functions on limited domains.	24. Find the inverse of $f(x) = 4x^2$ for $x \geq 0$. 25. Find the inverse of $g(x) = x^2 - 3$ for $x \geq 0$. 28. $f(x) = (x - 2)^2$ and $g(x) = \sqrt{x} + 2$ are inverse functions for what values of x? 29. Is the function $f(x) = 2x^3 + 1$ a one-to-one function? Does it have an inverse?
Graph inverse functions. Graphs of Inverse Functions The graphs of a function and its inverse are symmetric with respect to the line $y = x$.	26. Graph $g(x) = \sqrt{x}$ and its inverse $g^{-1}(x)$ for $x \geq 0$ on the same axes. 27. Graph $g(x) = \sqrt[3]{x}$ and its inverse $g^{-1}(x)$ on the same axes.

	30. Sketch the graph of $y = f^{-1}(x)$ on the axes with the graph of $y = f(x)$, shown below. 
Solve applications involving one-to-one and inverse functions.	32. Shoe Sizes If x is the size of a man's shoe in Britain, then $d(x) = x + 0.5$ is its size in the United States. - Find the inverse of the function. - Use the inverse function to find the British size of a shoe if it is U.S. size $8\frac{1}{2}$. (Source: Kuru International Exchange Association) 33. Shoe Sizes If x is the size of a man's shoe in the United States, then $t(x) = x + 34.5$ is its Continental size. - Find a function that will convert Continental shoe size to U.S. shoe size. - Use the inverse function to find the U.S. size if the Continental size of a shoe is 43. (Source: Kuru International Exchange Association)

	34. Investments If x dollars are invested at 10% for 6 years, the future value of the investment is given by $S(x) = x + 0.6x$. - Find the inverse of this function. - What do the outputs of the inverse function represent? - Use this function to find the amount of money that must be invested for 6 years at 10% to have a future value of \$24,000.
Answer questions involving inverse function graphs.	38. Body-Heat Loss The model for body-heat loss depends on the coefficient of convection $K = f(x)$, which depends on wind speed x according to the equation $f(x) = 4\sqrt{4x + 1}$. - What are the domain and range of this function without regard to the context of this application? - Find the inverse of this function. - What are the domain and range of the inverse function? 52. Path of a Ball If a ball is thrown into the air at a velocity of 96 feet per second from a building that is 256 feet high, the height of the ball after x seconds is $f(x) = 256 + 96x - 16x^2$ feet. - For how many seconds will the ball be in the air? - Is this function a one-to-one function over this time interval? - Give an interval over which the function is one-to-one. - Find the inverse of the function over the interval $0 \leq x \leq 3$. What does it give?

Chapter 5: Exponential and Logarithmic Functions

Chapter 5 Algebra Toolbox

Learning Outcomes and Notes	Examples
Simplify expressions involving integer exponents. For real numbers a and b and integers m and n, $a^m \cdot a^n = a^{m+n}$ (Product Property) $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ (Quotient Property) For real numbers a and b and integers m and n, $(ab)^m = a^m b^m$ (Power of a Product Property) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, \text{ for } b \neq 0$ (Power of a Quotient Property) $(a^m)^n = a^{mn}$ (Power of a Power Property) $a^{-m} = \frac{1}{a^m}, \text{ for } a \neq 0$	<i>In Exercises 1 and 2, use the properties of exponents to simplify.</i> $x^4 \cdot x^3$ $\frac{x^{12}}{x^7}$ $(4ay)^4$ $\left(\frac{3}{z}\right)^4$ $2^3 \cdot 2^2$ $(x^4)^2$ $y^5 \cdot y$ $\frac{w^{10}}{w^4}$ $(6bx)^3$ $\left(\frac{5z}{2}\right)^3$ $3^2 \cdot 3^3$ $(2y^3)^4$ <i>In Exercises 3–18, use the properties of exponents to simplify the expressions and remove all zero and negative exponents. Assume that all variables are nonzero.</i> 10^{5^0} 4^{2^2} $x^{-4} \cdot x^{-3}$ $y^{-5} \cdot y^{-3}$ $(c^{-6})^3$ $(x^{-2})^4$ $\frac{a^{-4}}{a^{-5}}$ $\frac{b^{-6}}{b^{-8}}$
Simplify expressions involving rational exponents.	$(x^{-1/2})(x^{2/3})$ $(y^{-1/3})(y^{2/5})$

	13. $(3a^{-3}b^2)(2a^2b^{-4})$ 14. $(4a^{-2}b^3)(-2a^4b^{-5})$ 15. $\left(\frac{2x^{-3}}{x^2}\right)^{-2}$ 16. $\left(\frac{3y^{-4}}{2y^2}\right)^{-3}$ 17. $\frac{28a^4b^{-3}}{-4a^6b^{-2}}$ 18. $\frac{36x^5y^{-2}}{-6x^6y^{-4}}$ <i>In Exercises 19–24, multiply or divide, as indicated, and simplify the result, assuming the expressions are real and the variables represent nonnegative real numbers.</i> 19. $\sqrt{3} \cdot \sqrt{27}$ 20. $\sqrt{2a} \cdot \sqrt{6a}$ 21. $\sqrt{8xy^3z} \cdot \sqrt{4x^2y^2z}$ 22. $\sqrt[3]{2xy} \cdot \sqrt[3]{4x^2y}$ 23. $\frac{\sqrt[3]{32}}{\sqrt[3]{4}}$ 24. $\frac{\sqrt{16a^3x}}{\sqrt{2ax}}$
Simplify radical expressions.	<i>In Exercises 25–29, combine the radicals, if possible.</i> 25. $2\sqrt{x} + 5\sqrt{x}$ 26. $5\sqrt[3]{ab} - 2\sqrt[3]{ab}$ 27. $3\sqrt{8} + 2\sqrt{2}$ 28. $3\sqrt{27} + 6\sqrt{3} - \sqrt{12}$ 29. $4x\sqrt[3]{xy} + \sqrt[3]{8x^4y}$

	<i>In Exercises 45–54, use the properties of exponents to simplify the expressions. Assume that all variables are nonzero.</i> 45. $x^{1/2} \cdot x^{5/6}$ 46. $y^{2/5} \cdot y^{1/4}$ 47. $(c^{2/3})^{5/2}$ 48. $(x^{3/2})^{3/4}$ 49. $\frac{x^{3/4}}{x^{1/2}}$ 50. $\frac{y^{3/8}}{y^{1/4}}$ 51. $\left(\frac{2b^{1/3}}{6c^{2/3}}\right)^3$ 52. $\left(\frac{9x^{2/3}}{16y^{5/6}}\right)^{3/2}$ 53. $(x^{4/9}y^{2/3}z^{-1/3})^{3/2}$ 54. $\left(\frac{-8x^9y^{1/2}}{27z^{3/2}}\right)^{4/3}$
Rationalize denominators.	<i>In Exercises 30–34, rationalize the denominator and simplify.</i> 30. $\frac{2}{\sqrt{10}}$ 31. $\frac{\sqrt{8}}{\sqrt{3}}$ 32. $\frac{5}{\sqrt[3]{2}}$ 33. $\frac{9}{3 + \sqrt{7}}$ 34. $\frac{6}{\sqrt{5} - \sqrt{3}}$

Convert numbers in standard notation to scientific notation and vice versa.	<i>In Exercises 35–38, write the numbers in scientific notation.</i> 35. 46,000,000 36. 862,000,000,000 37. 0.000094 38. 0.00000278 <i>In Exercises 39–42, write the numbers in standard form.</i> 39. 4.372×10^5 40. 7.91×10^6 41. 5.6294×10^{-4} 42. 6.3478×10^{-3}
Simplify expressions involving scientific notation.	<i>In Exercises 43 and 44, multiply or divide, as indicated, and write the result in scientific notation.</i> 43. $(6.250 \times 10^7)(5.933 \times 10^{-2})$ 44. $\frac{2.961 \times 10^{-2}}{4.583 \times 10^{-4}}$

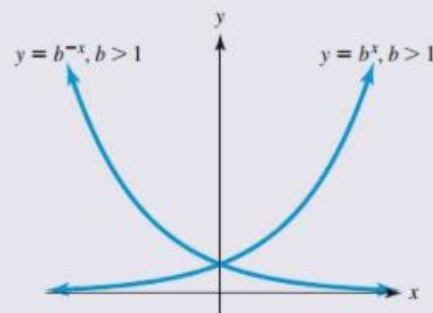
5.1 Exponential Functions

Learning Outcomes and Notes	Examples
Identify exponential functions and distinguish between exponential growth and decay. Exponential Function If b is a positive real number, $b \neq 1$, then the function $f(x) = b^x$ is an exponential function. The constant b is called the <i>base</i> of the function, and the variable x is the <i>exponent</i>.	1. Which of the following functions are exponential functions? a. $y = 3x + 3$ b. $y = x^3$ c. $y = 3^x$ d. $y = \left(\frac{2}{3}\right)^x$ e. $y = e^x$ f. $y = x^e$ 2. Determine if each of the following functions models exponential growth or exponential decay. a. $y = 2^{0.1x}$ b. $y = 3^{-1.4x}$ c. $y = 4e^{-5x}$ d. $y = 0.8^{3x}$

Graph exponential functions by plotting points or using a calculator.

Graphs of Exponential Functions

Equation: $y = b^x$
***x*-intercept:** none
***y*-intercept:** $(0, 1)$
Domain: all real numbers
Range: all real numbers $y > 0$
Horizontal asymptote: *x*-axis
Shape: an increasing function if $b > 1, y = b^x$
 a decreasing function if $y = b^x$ with $0 < b < 1$
 or if $y = b^{-x}$ with $b > 1$



3. a. Graph the function $f(x) = e^x$ on the window $[-5, 5]$ by $[-10, 30]$.

b. Find $f(1)$, $f(-1)$, and $f(4)$, rounded to three decimal places.

c. What is the horizontal asymptote of the graph?

d. What is the *y*-intercept?

4. a. Graph the function $f(x) = 5^x$ on the window $[-5, 5]$ by $[-10, 30]$.

b. Find $f(1)$, $f(3)$, and $f(-2)$.

c. What is the horizontal asymptote of the graph?

d. What is the *y*-intercept?

5. Graph the function $y = 3^x$ on $[-5, 5]$ by $[-10, 30]$.

6. Graph $y = 5e^{-x}$ on $[-4, 4]$ by $[-1, 20]$.

In Exercises 7–12, graph each function.

7. $y = 5^{-x}$

8. $y = 3^{-2x}$

9. $y = 3^x + 5$

10. $y = 2(1.5)^{-x}$

11. $y = 3^{(x-2)} - 4$

12. $y = 3^{(x-1)} + 2$

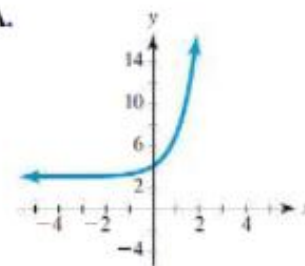
Use transformations to match a graph with its equation.

Vertical shift	$g(x) = b^x + k$	Graph is shifted k units up if k is positive, and $ k $ units down if k is negative.
Horizontal shift	$g(x) = b^{x-h}$	Graph is shifted $ h $ units right if h is positive, and $ h $ units left if h is negative.
Reflection across the x-axis	$g(x) = -b^x$	Graph is reflected across the x-axis.
Reflection across the y-axis	$g(x) = b^{-x}$	Graph is reflected across the y-axis.
Vertical stretch/compress	$g(x) = a \cdot b^x$	Graph is vertically stretched if $ a > 1$; graph is vertically compressed if $ a < 1$.
Horizontal stretch/compress	$g(x) = b^{a \cdot x}$	Graph is horizontally stretched if $ a < 1$; graph is horizontally compressed if $ a > 1$.

In Exercises 13–18, match the graph with its equation.

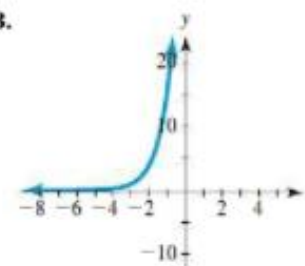
13. $y = 4^{x+3}$

A.



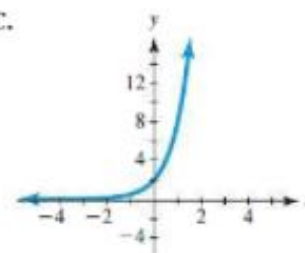
14. $y = 2 \cdot 4^x$

B.



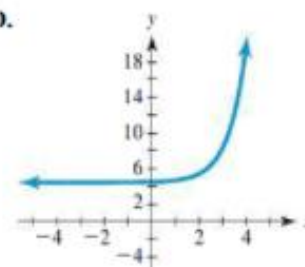
15. $y = 4^x + 3$

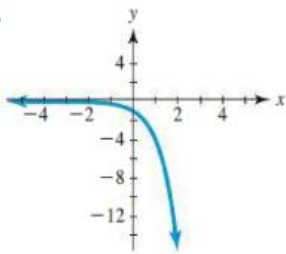
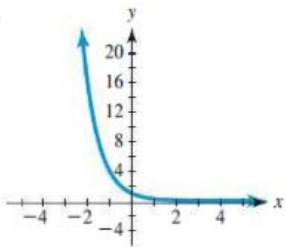
C.



16. $y = 4^{-x}$

D.



	17. $y = -4^x$ E.  18. $y = 4^{x-2} + 4$ F. 
Use transformations to graph exponential functions.	<i>In Exercises 19–24, use your knowledge of transformations to compare the graph of the function with the graph of $f(x) = 4^x$. Then graph each function for $-4 \leq x \leq 4$.</i> 19. $y = 4^x + 2$ 20. $y = 4^{x-1}$ 21. $y = 4^{-x}$ 22. $y = -4^x$ 23. $y = 3(4^x)$ 24. $y = 3 \cdot 4^{(x-2)} - 3$ <i>In Exercises 27–34, use your knowledge of transformations to compare the graph of each function with the graph of $f(x) = e^x$. Then graph each function.</i> 27. $y = e^{x-3}$ 28. $y = e^x - 4$ 29. $y = \frac{1}{2}e^x$ 30. $y = -e^x$ 31. $y = e^{-x}$ 32. $y = e^{x+2} + 1$ 33. $y = e^{3x}$ 34. $y = e^{(1/3)x}$

Solve applications involving exponential functions.

Exponential Growth Models

Equation: $y = a(b^{kx})$, $b > 1$, $a > 0$, $k > 0$

x-intercept: none

y-intercept: $(0, a)$

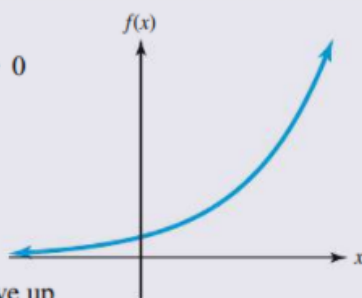
Domain: all real numbers

Range: all real numbers $y > 0$

Horizontal

asymptote: x-axis (the line $y = 0$)

Shape: increasing on domain and concave up



Exponential Decay Models

Equation: $y = a(b^{kx})$, $b > 1$, $a > 0$, $k < 0$

or

$y = a(b^{kx})$, $0 < b < 1$, $a > 0$, $k > 0$

x-intercept: none

y-intercept: $(0, a)$

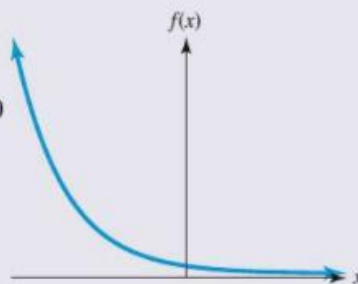
Domain: all real numbers

Range: $y > 0$

Horizontal

asymptote: x-axis (the line $y = 0$)

Shape: decreasing on domain and concave up



35. a. Graph $f(x) = 12e^{-0.2x}$ for $-5 \leq x \leq 15$.

b. Find $f(10)$ and $f(-10)$.

c. Does this function represent growth or decay?

37. **Sales Decay** At the end of an advertising campaign, weekly sales of smartphones declined according to the equation $y = 12,000(2^{-0.08x})$ dollars, where x is the number of weeks after the end of the campaign.

a. Determine the sales at the end of the ad campaign.

b. Determine the sales 6 weeks after the end of the campaign.

c. Does this model indicate that sales will eventually reach \$0?

41. **Continuous Compounding** If \$8000 is invested for t years at 8% interest, compounded continuously, the future value is given by $S = 8000e^{0.08t}$ dollars.

a. Graph this function for $0 \leq t \leq 15$.

b. Use the graph to estimate when the future value will be \$20,000.

44. **Radioactive Decay** A breeder reactor converts stable uranium-238 into the isotope plutonium-239. The decay of this isotope is given by $A(t) = 100e^{-0.00002876t}$, where $A(t)$ is the amount of the isotope at time t (in years) and 100 grams is the original amount.

a. How many grams remain after 100 years?

b. Graph this function for $0 \leq t \leq 50,000$.

c. The half-life is the time it takes for half of the initial amount to decay; use graphical methods to estimate the half-life of this isotope.

	50. Inflation An antique table increases in value according to the function $v(x) = 850(1.04^x)$ dollars, where x is the number of years after 1990. - How much was the table worth in 1990? - If the pattern indicated by the function remains valid, what was the value of the table in 2005? - Use a table or graph to estimate the year when this table would reach double its 1990 value. 54. Drugs in the Bloodstream If a drug is injected into the bloodstream, the percent of the maximum dosage that is present at time t is given by $y = 100(1 - e^{-0.35(10-t)})$ where t is in hours, with $0 \leq t \leq 10$. - What percent of the drug is present after 2 hours? - Graph this function. - When is the drug totally gone from the bloodstream?
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5.2 Logarithmic Functions; Properties of Logarithms

Learning Outcomes and Notes	Examples
Convert logarithmic expressions to exponential form. Logarithmic Function For $x > 0$, $b > 0$, and $b \neq 1$, the logarithmic function with base b is $y = \log_b x$ which is defined by $x = b^y$. That is, $\log_b x$ is the exponent to which we must raise the base b to get the number x. This function is the inverse function of the exponential function $y = b^x$.	<i>In Exercises 1–4, write the logarithmic equations in exponential form.</i> $y = \log_3 x$ $2y = \log_5 x$ $y = \ln(2x)$ $y = \log(-x)$

Natural Logarithms

The logarithmic function with base e is $y = \log_e x$, defined by $x = e^y$ for all positive numbers x and denoted by

$$\ln x = \log_e x$$

Convert exponential expressions to logarithmic form.

In Exercises 5–8, write the exponential equations in logarithmic form.

5. $x = 4^y$

6. $m = 3^p$

7. $32 = 2^5$

8. $9^{2x} = y$

Evaluate logarithms.

Basic Properties of Logarithms

For $b > 0, b \neq 1$,

$$1. \log_b b = 1$$

$$2. \log_b 1 = 0$$

$$3. \log_b b^x = x$$

$$4. \quad b^{\log_b x} = x$$

5. For positive real numbers M and N , if $M = N$, then $\log_b M = \log_b N$.

Additional Logarithmic Properties

For $b > 0$, $b \neq 1$, k a real number, and M and N positive real numbers,

$$6. \log_b(MN) = \log_b M + \log_b N \quad (\text{Product Property})$$

$$7. \log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N \quad (\text{Quotient Property})$$

8. $\log_b M^k = k \log_b M$ (Power Property)

In Exercises 9 and 10, evaluate the logarithms, if possible. Round each answer to three decimal places.

9. a. $\log 7$

10. a. $\log 456$

b. $\ln 86$

b. $\log(-12)$

c. $\log 63,980$

c. $\ln 10$

In Exercises 11–13, find the value of the logarithms without using a calculator.

11. a. $\log_2 32$

b. $\log_9 81$

c. $\log_3 27$

d. $\log_4 64$

e. $\log_5 625$

12. a. $\log_2 64$

b. $\log_9 27$

c. $\log_4 2$

d. $\ln(e^3)$

e. $\log 100$

13. a. $\log_3 \frac{1}{27}$

b. $\ln 1$

c. $\ln e$

d. $\log 0.0001$

In Exercises 23–26, use the properties of logarithms to evaluate the expressions.

23. $\log 10^{14}$

24. $\ln e^5$

25. $10^{\log_{10} 12}$

26. $6^{\log_6 25}$

In Exercises 27–30, use $\log_a(20) = 1.4406$ and $\log_a(5) = 0.7740$ to evaluate each expression.

27. $\log_a(100)$

28. $\log_a(4)$

29. $\log_a 5^3$

30. $\log_a \sqrt{20}$

Graph and analyze logarithmic functions.

Logarithmic Function

Equation: $y = \log_b x$

x-intercept: $(1, 0)$

y-intercept: none

Domain: $x > 0$

Range: all real numbers

Vertical

asymptote: y-axis (the line $x = 0$)

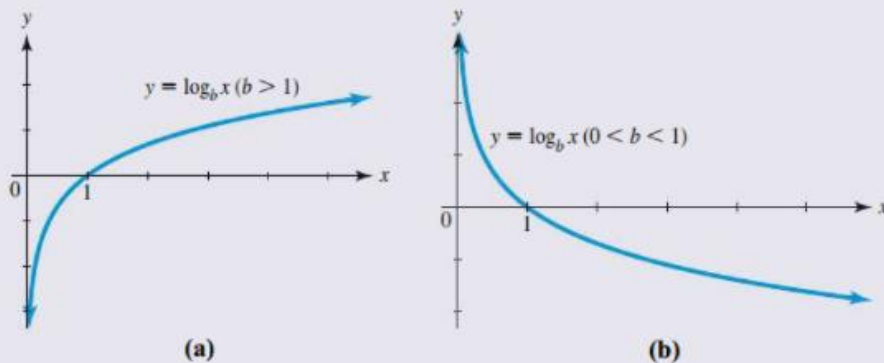


Figure 5.15

14. Graph the functions by changing to exponential form.

a. $y = \log_3 x$

b. $y = \log_5 x$

In Exercises 15–18, graph the functions.

15. $y = 2 \ln x$

16. $y = 4 \log x$

17. $y = \log(x + 1) + 2$

18. $y = \ln(x - 2) - 3$

Find and graph the inverse of exponential functions.	19. a. Write the inverse of $y = 4^x$ in logarithmic form. b. Graph $y = 4^x$ and its inverse and discuss the symmetry of their graphs. 20. a. Write the inverse of $y = 3^x$ in logarithmic form. b. Graph $y = 3^x$ and its inverse and discuss the symmetry of their graphs.
Rewrite logarithmic expressions using properties of logarithms.	<i>In Exercises 31–34, rewrite each expression as a sum, difference, or product of logarithms, and simplify if possible.</i> 31. $\ln \frac{3x - 2}{x + 1}$ 32. $\log [x^3(3x - 4)^5]$ 33. $\log_3 \frac{\sqrt[3]{4x + 1}}{4x^2}$ 34. $\log_3 \frac{\sqrt[3]{3x - 1}}{5x^2}$ <i>In Exercises 35–38, rewrite each expression as a single logarithm.</i> 35. $3 \log_2 x + \log_2 y$ 36. $\log x - \frac{1}{3} \log y$ 37. $4 \ln(2a) - \ln b$ 38. $6 \ln(5y) + 2 \ln x$

Solve applications involving logarithms.

Richter Scale

1. If the intensity of an earthquake is I , its Richter scale measurement is

$$R = \log \frac{I}{I_0}$$

2. If the Richter scale reading of an earthquake is k , its intensity is

$$I = 10^k I_0$$

3. If the difference of the Richter scale measurements of two earthquakes is the positive number d , the intensity of the larger earthquake is 10^d times that of the smaller earthquake.

39. Life Span On the basis of data and projections for the years 1920 through 2050, the expected life span of people in the United States can be described by the function $f(x) = 8.77 + 14.9 \ln x$ years, where x is the number of years from 1900 to the person's birth year.

- What does this model estimate the life span to the nearest year to be for people born in 1925?
- What does it predict the life span to the nearest year to be for people born in 2045?
- Explain why these numbers are so different.
(Source: National Center for Health Statistics)

45. Female Workers For the years 1970 to 2040, the percent of females over 15 years old in the workforce is given or projected by $y = 27.4 + 5.02 \ln x$, where x is the number of years from 1960.

- What does the model predict the percent to be in 2011? In 2015?
- Is the percent of female workers increasing or decreasing?

46. Obesity The percent of Americans who are obese from 1990 projected through 2030 can be modeled by $y = -31.732 + 18.672 \ln x$, with x equal to the number of years after 1980.

- Use the model to predict the percent of Americans who will be obese in 2032.
- Does this model predict that obesity will get better or worse in the near future?

49. Doubling Time The number of quarters (a quarter equals 3 months) needed to double an investment when a lump sum is invested at 8%, compounded quarterly, is given by $n = \frac{\log 2}{0.0086}$. In how many years will the investment double?

54. Earthquakes

- a. If an earthquake has an intensity of 250,000 times I_0 , what is the magnitude of the earthquake?
- b. Compare the magnitudes of two earthquakes when the intensity of one is 10 times the intensity of the other. (See Exercise 53.)

61. Japan Tsunami In May of 2008, an earthquake measuring 6.8 on the Richter scale struck near the east coast of Honshu, Japan. In March of 2011, an earthquake measuring 9.0 struck that same region, causing a devastating tsunami that killed thousands of people. The 2011 earthquake was how many times as intense as the one in 2008?

Use the following information to answer the questions in Exercises 62–67. To quantify the intensity of sound, the decibel scale (named for Alexander Graham Bell) was developed. The formula for loudness L on the decibel scale is $L = 10 \log \frac{I}{I_0}$, where I_0 is the intensity of sound just below the threshold of hearing, which is approximately 10^{-16} watt per square centimeter.

- 62. Decibel Scale** Find the decibel reading for a sound with intensity 20,000 times I_0 .
- 63. Decibel Scale** Compare the decibel readings of two sounds if the intensity of the first sound is 100 times the intensity of the second sound.

64. Decibel Scale Use the exponential form of the function $L = 10 \log \frac{I}{I_0}$ to find the intensity of a sound if its decibel reading is 40.

65. Decibel Scale Use the exponential form of $L = 10 \log \frac{I}{I_0}$ to find the intensity of a sound if its decibel reading is 140. (Sound at this level causes pain.)

To answer the questions in Exercises 68–72, use the fact that pH is given by the formula $\text{pH} = -\log[\text{H}^+]$.

68. pH Levels Find the pH value of beer for which $[\text{H}^+] = 0.0000631$.

69. pH Levels The pH value of eggs is 7.79. Find the hydrogen-ion concentration for eggs.

70. pH Levels The most common solutions have a pH range between 1 and 14. Use an exponential form of the pH formula to find the values of $[\text{H}^+]$ associated with these pH levels.

5.3 Exponential and Logarithmic Equations

Learning Outcomes and Notes	Examples
Solve exponential equations. Solving Exponential Equations Using Logarithmic Forms To solve an exponential equation using logarithmic form: 1. Rewrite the equation with the term containing the exponent by itself on one side. 2. Divide both sides by the coefficient of the term containing the exponent. 3. Change the new equation to logarithmic form. 4. Solve for the variable.	<i>In Exercises 1–10, solve the equations algebraically and check graphically. Round to three decimal places.</i> 1. $1600 = 10^x$ 2. $4600 = 10^x$ 3. $2500 = e^x$ 4. $54.6 = e^x$ 5. $8900 = e^{5x}$ 6. $2400 = 10^{8x}$ 7. $4000 = 200e^{8x}$ 8. $5200 = 13e^{12x}$ 9. $8000 = 500(10^x)$ 10. $9000 = 400(10^x)$
Evaluate logarithms using the change of base formula. Change-of-Base Formula If $b > 0, b \neq 1, a > 0, a \neq 1$, and $x > 0$, then $\log_a x = \frac{\log_b x}{\log_b a}$ In particular, for base 10 and base e, $\log_a x = \frac{\log x}{\log a} \quad \text{and} \quad \log_a x = \frac{\ln x}{\ln a}$	<i>In Exercises 11–14, use a change-of-base formula to evaluate each logarithm. Give your answers rounded to four decimal places.</i> 11. $\log_6 18$ 12. $\log_7 215$ 13. $\log_8 \sqrt{2}$ 14. $\log_4 \sqrt[3]{10}$

Solve logarithmic equations.	<i>In Exercises 15–22, solve the equations.</i> 15. $8^x = 1024$ 16. $9^x = 2187$ 17. $2(5^{3x}) = 31,250$ 18. $2(6^{2x}) = 2592$ 19. $5^{x-2} = 11.18$ 20. $3^{x-4} = 140.3$ 21. $18,000 = 30(2^{12x})$ 22. $5880 = 21(2^{3x})$ <i>In Exercises 23–36, solve the logarithmic equations.</i> 23. $\log_2 x = 3$ 24. $\log_4 x = -2$ 25. $5 + 2 \ln x = 8$ 26. $4 + 3 \log x = 10$ 27. $5 + \ln(8x) = 23 - 2 \ln x$ 28. $3 \ln x + 8 = \ln(3x) + 12.18$ 29. $2 \log x - 2 = \log(x - 25)$ 30. $\ln(x - 6) + 4 = \ln x + 3$ 31. $\log_3 x + \log_3 9 = 1$ 32. $\log_2 x + \log_2(x - 6) = 4$ 33. $\log_2 x = \log_2 5 + 3$ 34. $\log_2 x = 3 - \log_2 2x$ 35. $\log 3x + \log 2x = \log 150$ 36. $\ln(x + 2) + \ln x = \ln(x + 12)$
Solve exponential inequalities.	<i>In Exercises 37–40, solve the inequalities.</i> 37. $3^x < 243$ 38. $7^x \geq 2401$ 39. $5(2^x) \geq 2560$ 40. $15(4^x) \leq 15,360$

Solve applications involving exponential and logarithmic equations.

Solving Exponential Equations Using Logarithmic Properties

To solve an exponential equation using logarithmic properties:

1. Rewrite the equation with a base raised to a power on one side.
2. Take the logarithm, base e or 10, of both sides of the equation.
3. Use a logarithmic property to remove the variable from the exponent.
4. Solve for the variable.

47. Super Bowl Ads A minute ad during Super Bowl VII in 1973 cost \$200,000. The price tag for a 30-second ad slot during the 2011 Super Bowl was \$3 million. The cost of a 30-second slot of advertising for the Super Bowl can be modeled by $y = 0.0000966(1.101^x)$, where x is the number of years after 1900 and y is the cost in millions.

- a. According to the model, what was the cost of a 30-second Super Bowl ad in 2000?
- b. The cost of a 30-second ad during Super Bowl XLIX in 2015 was \$4.5 million. Does the model underestimate or overestimate the cost of these ads?

49. Purchasing Power The purchasing power (real value of money) decreases if inflation is present in the economy. For example, the purchasing power of \$40,000 after t years of 5% inflation is given by the model

$$P = 40,000e^{-0.05t} \text{ dollars}$$

How long will it take for the value of a \$40,000 pension to have a purchasing power of \$20,000 under 5% inflation?

(Source: Viewpoints, VALIC)

51. Real Estate Inflation During a 5-year period of constant inflation, the value of a \$100,000 property increases according to the equation $v = 100,000e^{0.03t}$ dollars. In how many years will the value of this building be double its current value?

54. Radioactive Decay The amount of radioactive isotope thorium-234 present in a certain sample at time t is given by $A(t) = 500e^{-0.02828t}$ grams, where t years is the time since the initial amount was measured. How long will it take for the amount of isotope to equal 318 grams?

55. Drugs in the Bloodstream The concentration of a drug in the bloodstream from the time the drug is injected until 8 hours later is given by

$$y = 100(1 - e^{-0.312(8-t)}) \text{ percent, } t \text{ in hours,}$$

where the drug is administered at time $t = 0$. In how many hours will the drug concentration be 79% of the initial dose?

58. Radioactive Decay A breeder reactor converts stable uranium-238 into the isotope plutonium-239. The decay of this isotope is given by $A(t) = A_0e^{-0.00002876t}$, where $A(t)$ is the amount of the isotope at time t (in years) and A_0 is the original amount. If the original amount is 100 pounds, find the half-life of this isotope.

62. Doubling Time The future value of a lump sum P that is invested for n years at 10%, compounded annually, is $S = P(1.10)^n$. Show that the number of years it would take for this investment to double is $n = \log_{1.10} 2$.

5.4 Exponential and Logarithmic Models

Learning Outcomes and Notes

Find a function that models given data.

Constant Percent Changes

If the percent change of the outputs of a set of data is constant for equally spaced inputs, an exponential function will be a perfect fit for the data.

If the percent change of the outputs is approximately constant for equally spaced inputs, an exponential function will be an approximate fit for the data.

Examples

- Find the exponential function that models the data in the table below.

x	-2	-1	0	1	2	3	4	5
y	2/9	2/3	2	6	18	54	162	486

- Find the exponential function that is the best fit for $f(x)$ defined by the table in Exercise 2.
- Find the exponential function that is the best fit for $h(x)$ defined by the table in Exercise 4.
- Find the exponential function that models the data in the table below. Round the model with three-decimal-place accuracy.

x	-2	-1	0	1	2	3
y	5	30	150	1000	4000	20,000

- Make a scatter plot of the data in the table below.
 - Does it appear that a linear model or a logarithmic model is the better fit for the data?

x	1	3	5	7	9
y	-2	1	3	4	5

Determine whether a function is perfectly, approximately, or not exponential.

2. The following table has input x and output $f(x)$. Test the percent change of the outputs to determine if the function is exactly exponential, approximately exponential, or not exponential.

x	1	2	3	4	5	6
$f(x)$	4	16	64	256	1024	4096

3. The following table has input x and output $g(x)$. Test the percent change of the outputs to determine if the function is exactly exponential, approximately exponential, or not exponential.

x	1	2	3	4	5	6
$g(x)$	2.5	6	8.5	10	8	6

4. The following table has input x and output $h(x)$. Test the percent change of the outputs to determine if the function is exactly exponential, approximately exponential, or not exponential.

x	1	2	3	4	5	6
$h(x)$	1.5	2.25	3.8	5	11	17

Determine the best model for given data.

7. a. Make a scatter plot of the data in the table below.
b. Does it appear that a linear model or an exponential model is the better fit for the data?

x	1	2	3	4	5	6
y	2	3.1	4.3	5.4	6.5	7.6

Exponential Model

If a set of data has initial value a and a constant percent change r (written as a decimal) for equally spaced inputs x , the data can be modeled by the exponential function

$$y = a(1 + r)^x$$

for exponential growth and by the exponential function

$$y = a(1 - r)^x$$

for exponential decay.

9. Compare the first differences and the percent change of the outputs to determine if the data in the table below should be modeled by a linear or an exponential function.

x	1	2	3	4	5
y	2	6	14	34	81

15. **a** Make a scatter plot of the data in the table below.
- b.** Find a power function that models the data. Round to three decimal places.
- c.** Find a quadratic function that models the data. Round to three decimal places.
- d.** Find a logarithmic function that models the data. Round to three decimal places.

x	1	2	3	4	5	6
y	3.5	5.5	6.8	7.2	8	9

16. Let $y = f(x)$ represent the power model found in Exercise 15(b), $y = g(x)$ represent the quadratic model found in Exercise 15(c), and $y = h(x)$ represent the logarithmic model found in Exercise 15(d). Graph each function on the same axes as the scatter plot, using the window $[0, 12]$ by $[0, 15]$. Which model appears to be the best fit?

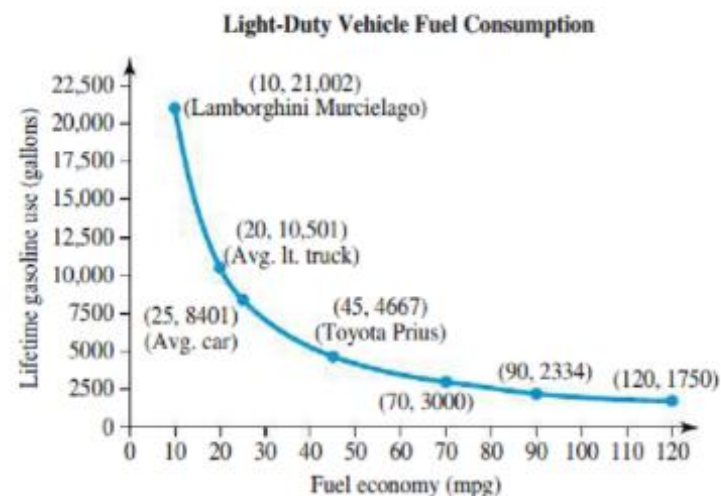
Solve applications involving exponential and logarithmic models.

Use the exponential form $y = a(1 + r)^x$ to model the information in Exercises 17–20.

- 17. Inflation** Suppose that the retail price of an automobile is \$30,000 in 2000 and that it increases at 4% per year.
- Write the equation of the exponential function that models the retail price of the automobile t years after 2000.
 - Use the model to predict the retail price of the automobile in 2015.
- 18. Population** Suppose that the population of a city is 190,000 in 2000 and that it grows at 3% per year.
- Write the equation of the exponential function that models the annual growth.
 - Use the model to find the population of this city in 2010.
- 21. Personal Income** The table shows the total personal income in the United States (in billions of dollars) for selected years from 1960 and projected to 2024.
- Find the best-fitting exponential function that will model the data, with x equal to the number of years after 1960 and y in billions of dollars. Report the model with four significant digits.
 - If the model is valid in 2030, what is the projected total personal income then?

35. Fuel Economy The lifetime gasoline use of light-duty vehicles is a function of the fuel economy, as shown in the figure below.

- Should the data be modeled by an exponential growth or an exponential decay function?
- Find an exponential function to model the data.



(Source: U.S. Department of Energy, Energy Efficiency, and Renewable Energy)

- Find a power function to model the data.
- Which function is the better fit to the data?
- If a vehicle had a fuel economy of 100 mpg, what would its lifetime gasoline use be, according to the power model?

Answer questions involving exponential growth models.

29. Rate of Growth of Population The table gives the annual rates of growth of the U.S. population for selected years from 1970 projected to 2049.

- a. Find a logarithmic function that models the rate, y , as a function of the number of years after 1960.
- b. Find an exponential function that models the rate, y , as a function of the number of years after 1960.
- c. Graph each model on the same axes with the data to determine which model is the better fit to the data.

Year	Annual Growth Rate
1970	2.07
1980	1.69
1990	1.56
2000	1.25
2010	1.10
2020	0.90
2030	0.76
2040	0.62
2049	0.46

	38. Decomposition of Chlorine in a Pool Under certain water conditions, the free chlorine in a swimming pool decomposes according to the equation $C(t) = C_0 e^{kt}$. Initially, the amount of free chlorine in a pool was 2.8 parts per million (ppm). Eighteen hours later, the amount was 2.1 ppm. a. What is the rate of decay of the amount of free chlorine in the pool? b. When the chlorine level reaches 1.0 ppm, the pool must be shocked. How long will it be before the pool must be shocked?
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5.6 Annuities; Loan Repayment

Learning Outcomes and Notes	Examples
Solve annuity equations for specific variables.	1. Solve $S = P(1 + i)^n$ for P with positive exponent n. 2. Solve $S = P(1 + i)^n$ for P, with no denominator. 3. Solve $Ai = R[1 - (1 + i)^{-n}]$ for A. 5. Solve $A = R\left(\frac{1 - (1 + i)^{-n}}{i}\right)$ for R.
Find the future value of an ordinary annuity. Future Value of an Ordinary Annuity If R dollars are contributed at the end of each period for n periods into an annuity that pays interest at rate i at the end of the period, the future value of the annuity is $S = R\left(\frac{(1 + i)^n - 1}{i}\right) \text{ dollars}$	7. IRA Anne Wright decides to invest \$4000 in an IRA CD at the end of each year for 10 years. If she makes these payments and the certificates all pay 6%, compounded annually, how much will she have at the end of the 10 years? 8. Future Value Find the future value of an annuity of \$5000 paid at the end of each year for 20 years, if interest is earned at 9%, compounded annually.

	10. Future Value Find the future value of an annuity of \$2600 paid at the end of each 3-month period for 5 years, if interest is earned at 6%, compounded quarterly. 11. Retirement Mr. Lawrence invests \$600 at the end of each month in an account that pays 7%, compounded monthly. How much will be in the account in 25 years? 12. Down Payment To accumulate money for the down payment on a house, the Kings deposit \$800 at the end of each month into an account paying 7%, compounded monthly. How much will they have at the end of 5 years?
Find the present value of an ordinary annuity. Present Value of an Ordinary Annuity If a payment of R dollars is to be made at the end of each period for n periods from an account that earns interest at a rate of i per period, then the account is an ordinary annuity, and the present value is $A = R \left(\frac{1 - (1 + i)^{-n}}{i} \right)$	4. Evaluate $2000 \left(\frac{1 - (1 + 0.01)^{-240}}{0.01} \right)$. 6. Evaluate $240,000 \left(\frac{0.01}{1 - (1 + 0.01)^{-120}} \right)$. 15. Annuities Find the present value of an annuity that will pay \$1000 at the end of each year for 10 years if the interest rate is 7%, compounded annually. 16. Annuities Find the present value of an annuity that will pay \$500 at the end of each year for 20 years if the interest rate is 9%, compounded annually.

Find the payments needed to amortize a loan. Amortization Formula If a debt of A dollars, with interest at a rate of i per period, is amortized by n equal periodic payments made at the end of each period, then the size of each payment is $R = A \left(\frac{i}{1 - (1 + i)^{-n}} \right)$	26. <i>Loan Repayment</i> A loan of \$36,000 is to be amortized with monthly payments over 6 years. If the interest on the loan is 6% per year, paid on the unpaid balance, - What is the interest rate charged each month on the unpaid balance? - How many payments are made to repay the loan? - What payment is required each month to amortize the loan?
Solve applications involving annuities and loan repayments.	21. <i>Business Sale</i> A man can sell his Thrifty Electronics business for \$800,000 cash or for \$100,000 plus \$122,000 at the end of each year for 9 years. - Find the present value of the annuity that is offered if money is worth 10%, compounded annually. - If he takes the \$800,000, spends \$100,000 of it, and invests the rest in a 9-year annuity at 10%, compounded annually, what size annuity payment will he receive at the end of each year? - Which is better, taking the \$100,000 and the annuity or taking the cash settlement? Discuss the advantages of your choice. 24. <i>Auto Loan</i> A man wants to buy a car and can afford to pay \$400 per month. - If he can get a loan for 48 months with interest at 12% per year on the unpaid balance and make monthly payments, how much can he pay for a car? - What is the total amount paid over the life of the loan? - What is the total interest paid on the loan?